How can we factor polynomials?

Factoring refers to writing something as a product.

Factoring completely means that all of the factors are relatively prime (they have a GCF of 1).

Methods of factoring:

1. Greatest Common Factor (GCF) - Any polynomial
2. Grouping - Only for 4 or 6 term polynomials
3. Trinomial Method - Only for trinomials
4. Speed Factoring - Special cases only

Method 1: Factoring Out the Greatest Common Factor (GCF)

Factoring out the GCF can be done by using the distributive property.

Ex 1: Factor $12x^3 + 3x^2$.

Step 1: Find the GCF of $12x^3$ and $3x^2$.

The GCF is $3x^2$.

Step 2: Rewrite by factoring out the GCF.

$$3x^2(4x+1)$$

Method 2: Factoring by Grouping

Ex 1: $12xy+20x+9y+15$

$(12xy+20x)+(9y+15)$

$4x(3y+5)+3(3y+5)$

$\boxed{(3y+5)(4x+3)}$

Step 1: Group terms together that have a common monomial factor.

Step 2: Factor out the GCF of each group.

Step 3: Find the common polynomial factor and factor it out using the distributive property.

Ex 2: $6xy+8x-21y-28$

$(6xy+8x)+(-21y-28)$

$2x(3y+4)+(-7)(3y+4)$

$\boxed{(3y+4)(2x-7)}$

Ex 3: $4x^2z^2-10x^2-6yz+8yz^2-3x^2z-20y$

$(4x^2z^2-3x^2z-10x^2)+(8yz^2-6yz-20y)$

$x^2(4z^2-3z-10)+2y(4z^2-3z-10)$

$\boxed{(4z^2-3z-10)(x^2+2y)}$

Method 3: Factoring Using the Trinomial Method

Step 1: Write the trinomial in descending order.

Step 2: Find two numbers whose product is the same as the product of the first and third coefficients and whose sum is equal to the middle coefficient. (Make a chart.)

Step 3: Rewrite the middle term as the sum of two terms.

Step 4: Use the distributive property and factor by grouping.

Ex 1: $2x^2 - 5x - 3$

$(2x^2 - 6x) + (x - 3)$

$2x(x-3) + 1(x-3)$

$\boxed{(x-3)(2x+1)}$

$\times(-6)$	$+(-5)$
$-6(1)$	✓

Ex 2: $20y^2 + 13yz + 2z^2$

$(20y^2 + 8yz) + (5yz + 2z^2)$

$4y(5y+2z) + z(5y+2z)$

$\boxed{(5y+2z)(4y+z)}$

$\times(40)$	$+(13)$
$8(5)$	

Method 4: Speed Factoring - Special Cases

I. The Difference of Squares

II. Trinomials with a lead coefficient of 1

Special Case: The Difference of Squares

Consider the product: $(a+b)(a-b)$

$a^2 - ab + ab - b^2$

$a^2 - b^2$

☆ Since $(a+b)(a-b) = a^2 - b^2$, then $a^2 - b^2 = (a+b)(a-b)$.

☆ $a^2 - b^2$ is called the "difference of squares."

Ex 1: $x^2 - 121$

$(x)^2 - (11)^2$

$\boxed{(x-11)(x+11)}$

$x^2 + 11x - 11x - 121$

Ex 2: $25x^2 - 1$

$(5x)^2 - (1)^2$

$\boxed{(5x-1)(5x+1)}$

Ex 3: $72y^2 - 50$

$2(36y^2 - 25)$

$2[(6y)^2 - (5)^2]$

$\boxed{2(6y-5)(6y+5)}$

Ex 4: $m^4 - 16$

$(m^2)^2 - (4)^2$

$(m^2-4)(m^2+4)$

$[(m)^2-(2)^2](m^2+4)$

$\boxed{(m-2)(m+2)(m^2+4)}$

Special Case: Trinomials with a lead coefficient of 1

$$x^2 + bx + c$$

Find the two numbers whose product is c and whose sum is b.
These are the two numbers in the binomials.

Ex 1: $x^2 + 2x + 1$

$\boxed{(x+1)(x+1)}$

Ex 2: $x^2 - x - 12$

$\times(-12)$	$+(-1)$
$-4(3)$	✓

$\boxed{(x-4)(x+3)}$

Ex 3: $x^2 - 10x + 16$

$\boxed{(x-8)(x-2)}$

Ex 4: $x^2 + 28x + 160$

$\boxed{(x+20)(x+8)}$

Solving Equations by Factoring - Using the Zero Product Property

The Zero Product Property:

If xy = 0, then either x = 0 or y = 0.

Use the zero product property to solve the following equations.

Ex 1: $x(x-1)=0$

$x=0$ or $x-1=0$

$x=1$

$\boxed{x=0,1}$

Ex 2: $(x-5)(x+2)=0$

$x-5=0$ or $x+2=0$

$x=5$ or $x=-2$

$\boxed{x=-2,5}$

Ex 3: $5x(x-4)=0$

$5x=0$ or $x-4=0$

$x=0$ or $x=4$

$\boxed{x=0,4}$

Geometry 1.0 Algebra Review Notes - Solving Quadratic Equations Part II - Key

If the polynomial is not "set equal to zero", get all of the terms on one side of the equation first. Then factor the polynomial before trying to use the zero product property to solve.

Ex 4: $x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$(x^2 - 5x) + (2x - 10) = 0$

$x(x-5) + 2(x-5) = 0$

$(x-5)(x+2) = 0$

$x-5=0$ or $x+2=0$

$x = -2, 5$

Ex 5: $18 - 3x = x^2$

$0 = x^2 + 3x - 18$

$0 = (x^2 - 3x) + (6x - 18)$

$0 = x(x-3) + 6(x-3)$

$0 = (x-3)(x+6)$

$x-3=0$ or $x+6=0$

$x = -6, 3$

Ex 6: $w^3 - w^2 = 4w - 4$

$(w^3 - w^2) + (-4w + 4) = 0$

$w^2(\underline{w-1}) + (-4)(\underline{w-1}) = 0$

$(w-1)(w^2-4) = 0$

$(w-1)(w-2)(w+2) = 0$

$\boxed{w = -2, 1, 2}$

Ex 7: $m^3 = 121m$

$m^3 - 121m = 0$

$m(m^2 - 121) = 0$

$m(m-11)(m+11) = 0$

$\boxed{m = -11, 0, 11}$

Factoring Special Products

Old: Difference of Squares

$$a^2 - b^2 = (a-b)(a+b)$$

New: Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

Factor each expression completely.

Ex 1: $h^2 + 4h + 4$

$(h)^2 + 2(2)(h) + (2)^2$

$\boxed{(h+2)^2}$

Ex 2: $n^2 - 12n + 36$

$(n)^2 - 2(6)(n) + (6)^2$

$\boxed{(n-6)^2}$

Ex 3: $2y^2 - 20y + 50$

$2(y^2 - 10y + 25)$

$2[(y)^2 - 2(5)(y) + (5)^2]$

$\boxed{2(y-5)^2}$

Ex 4: $3x^2 + 6xy + 3y^2$

$3(x^2 + 2xy + y^2)$

$\boxed{3(x+y)^2}$

Solve each equation.

Ex 5: $a^2 + 6a + 9 = 0$

$(a)^2 + 2(3)(a) + (3)^2 = 0$

$(a+3)^2 = 0$

$a + 3 = 0$

$a = -3$

Ex 6: $w^2 - 14w + 49 = 0$

$(w)^2 - 2(7)(w) + (7)^2 = 0$

$(w-7)^2 = 0$

$w - 7 = 0$

$w = 7$

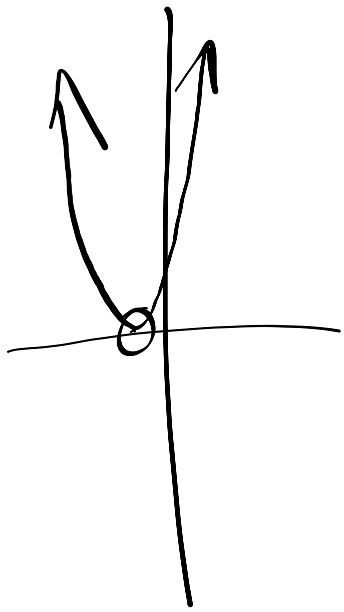
Ex 7: $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$

$$(x)^2 + 2\left(\frac{1}{3}\right)(x) + \left(\frac{1}{3}\right)^2 = 0$$

$$\left(x + \frac{1}{3}\right)^2 = 0$$

$$x + \frac{1}{3} = 0$$

$$\boxed{x = -\frac{1}{3}}$$



Solving Quadratic Equations by Completing the Square

What can be added to each polynomial so that the expression becomes a square of a binomial?

$x^2 + 8x + \boxed{16}$	$x^2 - 12x + \boxed{36}$	$x^2 + 3x + \boxed{\frac{9}{4}}$
$(x)^2 + 2(4)(x) + (4)^2$	$(x)^2 - 2(6)(x) + (6)^2$	$(x)^2 + 2\left(\frac{3}{2}\right)(x) + \left(\frac{3}{2}\right)^2$
$(x+4)^2$	$(x-6)^2$	$\left(x + \frac{3}{2}\right)^2$

- To complete the square for the expression $x^2 + bx$, add the square of half the coefficient of the bx term.

Solve each quadratic equation by completing the square.

Ex 1: $x^2 - 2x = 3$

$$(x)^2 - 2(1)(x) + \underline{\underline{(1)^2}} = 3 + \underline{\underline{(1)^2}}$$

$$(x-1)^2 = 4$$

$$x - 1 = \pm\sqrt{4}$$

$$x = 1 \pm 2$$

$$\boxed{x = -1, 3}\ \checkmark$$

Ex 2: $m^2 + 10m = -8$

$$(m)^2 + 2(5)(m) + \underline{\underline{(5)^2}} = -8 + \underline{\underline{(5)^2}}$$

$$(m+5)^2 = 17$$

$$m + 5 = \pm\sqrt{17}$$

$$\boxed{m = -5 \pm \sqrt{17}}$$

Geometry 1.0 Algebra Review Notes - Solving Quadratic Equations Part II - Key

Ex 3: $\frac{3h^2 - 24h + 27}{3} = \frac{0}{3}$

$h^2 - 8h + 9 = 0$

$\quad\quad\quad -9 \quad -9$

$(h)^2 - 2(4)(h) + (4)^2 = -9 + (4)^2$

$(h-4)^2 = 7$

$h - 4 = \pm\sqrt{7}$

$h = 4 \pm \sqrt{7}$

Ex 4: $\frac{3x^2 - 8x - 10}{3} = \frac{0}{3}$

$x^2 - \frac{8}{3}x - \frac{10}{3} = 0 \quad\quad \frac{30}{9} \quad \frac{16}{9}$

$\quad\quad + \frac{10}{3} \quad + \frac{10}{3}$

$x^2 - \frac{8}{3}x = \frac{10}{3}$

$(x)^2 - 2\left(\frac{4}{3}\right)(x) + \left(\frac{4}{3}\right)^2 = \frac{10}{3} + \left(\frac{4}{3}\right)^2$

$\left(x - \frac{4}{3}\right)^2 = \frac{46}{9}$

$x - \frac{4}{3} = \pm\sqrt{\frac{46}{9}}$

$x = \frac{4}{3} \pm \frac{\sqrt{46}}{3}$

$\boxed{x = \frac{4 \pm \sqrt{46}}{3}}$

Ex 5: $\dfrac{ax^2+bx+c=0}{a \qquad\qquad a}$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$\quad -\frac{c}{a} \quad -\frac{c}{a}$

$-\frac{4ac}{4a^2} \qquad \frac{b^2}{4a^2}$

$(x)^2+2\left(\frac{b}{2a}\right)(x)+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

☆ This is a proof of the Quadratic Formula!

Solving Quadratic Equations Using the Quadratic Formula

For any quadratic equation $0=ax^2+bx+c$,

the solution(s) are $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Step 1: Write the equation in standard form.
(Set equal to zero and in descending order)

Step 2: Identify all coefficients. a = ____, b = ____, c = ____.

Step 3: Substitute a, b, and c into the formula and simplify.

Geometry 1.0 Algebra Review Notes - Solving Quadratic Equations Part II - Key

Solve each equation using the quadratic formula.
Circle your final answer and use SRF if necessary.

Ex 1: $x^2 + 8x - 1 = 0$

$a = 1$

$b = 8$

$c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{-8 \pm \sqrt{64 + 4}}{2}$

$x = \frac{-8 \pm \sqrt{68}}{2}$

$x = \frac{-8 \pm 2\sqrt{17}}{2}$

$\boxed{x = -4 \pm \sqrt{17}}$

Ex 2: $x^2 + 6x = 5$

$x^2 + 6x - 5 = 0$

$a = 1$

$b = 6$

$c = -5$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-5)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{36 + 20}}{2}$

$x = \frac{-6 \pm \sqrt{56}}{2}$

$x = \frac{-6 \pm 2\sqrt{14}}{2}$

$\boxed{x = -6 \pm \sqrt{14}}$

Ex 3: $3n^2 = 5n - 1$

$3n^2 - 5n + 1 = 0$

$a = 3$

$b = -5$

$c = 1$

$n = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$

$n = \frac{5 \pm \sqrt{25 - 12}}{6}$

$\boxed{n = \frac{5 \pm \sqrt{13}}{6}}$

Ex 4: $5x^2 + 12x + 10 = 9 + 9x$

$5x^2 + 3x + 1 = 0$

$a = 5$

$b = 3$

$c = 1$

$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(1)}}{2(5)}$

$x = \frac{-3 \pm \sqrt{9 - 20}}{10}$

$x = \frac{-3 \pm \sqrt{-11}}{10}$ *

No Real Solution